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EFFECT OF RAREFACTION ON THE PROCESSES OF GASDYNAMIC FRICTION DURING BLOWING OR SUCTION THROUGH THE WALL ADJACENT TO THE FLOW

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From *Inzhenerno-Fizicheskiy Zhurnal,*
Vol. 7, No. 6, 1964

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1966

ASA TT F-406

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Translation of "K voprosu o vliyanii razrezheniya na protsessy gazodinamicheskogo treniya pri nalichii vduva ili otsosa gaza cherez omyvayemuyu potokom stenku." Inzhenerno-Fizicheskiy Zhurnal, Vol. 7, No. 6, pp. 48-54, 1964.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Yu. A. Koshmarov

ABSTRACT

The author discusses the problem of the flow of a rarefied gas between two porous plates that move at a small relative velocity. The analysis is based on the molecular concepts of a gas. Expressions are derived for calculating the velocity fields and fields of frictional stresses; these expressions are shown to be valid for any degree of rarefaction.

The problem of gasdynamic friction in the presence of transverse mass flow away from the surface adjoining the flow of a highly rarefied gas, when the Navier-Stokes continuity equations are not applicable, is of interest in several fields of technology. The object of the present study is the investigation of the mechanisms of these processes as they apply to the simple case of Couette flow. /48*

System of Equations and Initial Conditions

The problem of gas flow between two permeable parallel plates, one fixed, and the other moving with a constant velocity u_0, is considered.

An orthogonal xyz coordinate system is used and is considered to be fixed with respect to the stationary plate (the y-axis is perpendicular to the plates, the xz plane coincides with the surface of the stationary plate).

The following assumptions are used in the analysis: (a) the velocity of the moving plate is small compared with the speed of sound in the gas (i.e., $M_1 \ll 1$); (b) the transverse flux of the gas mass per unit surface area of both plates, is everywhere equal and time-invariant; (c) the transverse gas velocity (in the direction perpendicular to the plates) is small in comparison with the speed of sound in the gas ($M_2 \ll 1$); (d) mass forces are absent; (e) the gas

*Numbers given in the margin indicate the pagination in the original foreign text.

molecules repel each other according to the fifth power of their distance of separation (Maxwell molecules), and interact with the plate to provide for total diffusion reflection; (f) the temperature of both plates is the same everywhere, and is equal to T.

Several gas flow regions are distinguished depending on the ratio of the mean free path to some significant flow dimension L. The state of free molecule flow and the state defined by the Navier-Stokes equations are the two limiting cases. The methods of dynamic analysis for the two cases differ.

In order to describe the gas flow in both states from a single point of view, it is necessary to proceed either from the Maxwell-Boltzmann equation for the distribution function (ref. 1), or from an infinite system of moment /49 equations, derived from and equivalent to the Maxwell-Boltzmann equation (ref. 2). Since in the majority of gasdynamic problems, the macroscopic flow parameters are of prime interest (gas velocity, friction stresses, density, etc.) and form the lower moments of the molecular velocity distribution function (or functions of these Moments), it is expedient to utilize the system of moment equations.

In this case, the moment equation has the form (ref. 1)

$$\frac{d}{dy}\left(\int \xi_y Q f d\xi\right) = \Delta Q, \tag{1}$$

where

$$\Delta Q = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{+\infty}\int_{0}^{2\pi} (Q' - Q) f f_1 d\xi\, d\xi_1\, db\, d\varepsilon;\; Q = Q(\xi_x, \xi_y, \xi_z).$$

The important advantage of using the moment equations is a wide flexibility in the choice of the unknown velocity distribution function. Following reference 3 we represent the distribution function as follows:

for molecules with $\xi_y < 0$

$$f_1 = \frac{n_1}{(2\pi RT_1)^{3/2}} \exp\left[-\frac{(\xi_x - u_1)^2 + (\xi_y - v_1)^2 + \xi_z^2}{2RT_1}\right]; \tag{2}$$

for molecules with $\xi_y > 0$

$$f_2 = \frac{n_2}{(2\pi RT_2)^{3/2}} \exp\left[-\frac{(\xi_x - u_2)^2 + (\xi_y - v_2)^2 + \xi_z^2}{2RT_2}\right]. \tag{3}$$

The quantities n_1, n_2, T_1, T_2, u_1, u_2, v_1, v_2 are unknown functions of the coordinate y. By using equations (1) and (2) the total space moments can be

expressed in the following way in terms of the semispace moments

$$\int \Phi f d\xi = \int_{-\infty}^{+\infty}\int_{-\infty}^{0}\int_{-\infty}^{+\infty} \Phi f_1 d\xi_x d\xi_y d\xi_z + \int_{-\infty}^{+\infty}\int_{0}^{+\infty}\int_{-\infty}^{+\infty} \Phi f_2 d\xi_x d\xi_y d\xi_z. \tag{4}$$

In view of the assumptions made above concerning the velocity of the moving plate and the transverse gas velocity, as well as the temperature equilibrium of both plates, we can neglect the relative difference in temperatures and gas densities at different points in the flow, i.e., $T_1 = T_2 = T$; $n_1 = n_2 = n$. Thus when (2) and (3) are expanded in series we can neglect all the powers higher than the first relative to M. In this case instead of expressions (2) and (3) we have

for $\xi_y < 0$

$$f_1 = \frac{n}{(2\pi RT)^{3/2}}\left[1 + \frac{\xi_x u_1}{RT} + \frac{\xi_y v_1}{RT}\right] \exp\left(-\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT}\right); \tag{5}$$

for $\xi_y > 0$

$$f_2 = \frac{n}{(2\pi RT)^{3/2}}\left[1 + \frac{\xi_x u_2}{RT} + \frac{\xi_y v_2}{RT}\right] \exp\left(-\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT}\right). \tag{6}$$

In this case the problem is reduced to the determination of velocity fields /50
and fields of frictional stresses in the gas.

The values for the macroscopic flow parameters--velocities and the frictional stresses--may be expressed in a dimensionless form with the aid of (4), (5) and (6) in the following way

$$\bar{v} = \frac{1}{2}(\bar{v}_1 + \bar{v}_2); \tag{7}$$

$$\bar{u} = \frac{1}{2}(\bar{u}_1 + \bar{u}_2); \tag{8}$$

$$\bar{P}_{xy} = -[(\bar{u}_2 - \bar{u}_1) - \sqrt{2\pi\gamma}\, M_1 \bar{u}\bar{u}]. \tag{9}$$

The unknown functions $\bar{u}_1$, $\bar{u}_2$, $\bar{v}_1$, $\bar{v}_2$ are determined by assuming that (5) and (6) must satisfy (1) when $Q = m$, $Q = m\xi_x$, $Q = m\xi_y$ and $Q = m\xi_x\xi_y$. When $Q = m$ and $Q = m\xi_y$ equation (1),[1] taking into account (4), (5), (6) and condition (b),

[1]With $Q = m$ and $Q = m\xi_y$, the right part of (1) is equal to zero.

gives simple relationships, which in conjunction with the boundary conditions for $\overline{v}_2$ (with $\overline{y} = 0$ $\overline{v}_2 = \overline{j}$; with $\overline{y} = 1$ $\overline{v}_2 = \overline{j}$) leads to the following result

$$\overline{v}_1 = \overline{v}_2 = \overline{v} = \overline{j}. \tag{10}$$

Assuming that $Q = m\xi_x$ and $Q = m\xi_x\xi_y$, we obtain from (1) the following two equations[1]

$$\overline{u}_2 - \overline{u}_1 = \alpha, \tag{11}$$

$$\frac{d}{d\overline{y}}\left(\frac{\overline{u}_2 + \overline{u}_1}{2}\right) + \frac{\mathrm{Re}}{M_1}\left[\frac{\alpha}{\sqrt{2\pi\gamma}} - M_1\overline{j}\left(\frac{\overline{u}_2 + \overline{u}_1}{2}\right)\right] = 0, \tag{12}$$

where α is the integration constant.

The boundary conditions for $\overline{u}_1$ and $\overline{u}_2$ are

$$\begin{aligned} &\text{with } \overline{y} = 0 \quad \overline{u}_2 = 0; \\ &\text{with } \overline{y} = 1 \quad \overline{u}_1 = 1. \end{aligned} \tag{13}$$

The system of equations (11) and (12) with the boundary conditions (13) is sufficient to determine the spatial behavior of functions $\overline{u}_1$ and $\overline{u}_2$.

Solution of Equations and the Analysis of Results

Taking into account (13), let us write (12) in the following form:

$$\frac{d\overline{u}_2}{d\overline{y}} - \frac{\mathrm{Re}}{M_1}M_2\overline{u}_2 + \frac{\mathrm{Re}}{M_1}\,\alpha\left[\frac{1}{\sqrt{2\pi\gamma}} + \frac{M_2}{2}\right] = 0. \tag{14}$$

Relationship (14) is a first order linear equation in $\overline{u}_2$. Its solution /51
with boundary conditions (13) is of the form

$$\overline{u}_2 = \frac{\alpha}{M_2}\left[\frac{1}{\sqrt{2\pi\gamma}} + \frac{M_2}{2}\right]\left[1 - \exp\left(\frac{\mathrm{Re}}{M_1}M_2\overline{y}\right)\right]. \tag{15}$$

[1]With $Q = m\xi_x$ we have: $\Delta Q = 0$; with $Q = m\xi_x\xi_y$ we have $\Delta Q = \frac{mnRT}{\mu} P_{xy}$,

$\mu = \frac{kT}{(3/2)A_2(2mK)^{1/2}}$ (ref. 1).

Analyzing (11) and (15) together, and with boundary conditions (13) we obtain

$$\alpha = \left\{\left[1 - \exp\left(\frac{Re}{M_1} M_2\right)\right]\left[\frac{1}{M_2}\left(\frac{1}{\sqrt{2\pi\gamma}} + \frac{M_2}{2}\right)\right] - 1\right\}^{-1}. \tag{16}$$

From (8), (9), (11), (15) and (16) we obtain the following equations for the gas velocity $\bar{u}$ and frictional stress $\bar{P}_{xy}$

$$\bar{u} = \frac{\left[\exp\left(\frac{Re}{M_1} M_2 \bar{y}\right) - 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{1}{M_2} + 1\right] + 1}{\left[\exp\left(\frac{Re}{M_1} M_2\right) - 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{1}{M_2} + 1\right] + 2}, \tag{17}$$

$$\bar{P}_{xy} = \frac{2\left[1 + \left(\frac{\pi\gamma}{2}\right)^{1/2} M_2\right] \exp\left(\frac{Re}{M_1} M_2 \bar{y}\right)}{\left[\exp\left(\frac{Re}{M_1} M_2\right) - 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{1}{M_2} + 1\right] + 2}. \tag{18}$$

If $M_2 \to 0$ (the case when there is no transverse mass flow) equations (17) and (18) are reduced to the form

$$\bar{u} = \left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{Re}{M_1} \bar{y} + 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{Re}{M_1} + 2\right]^{-1}, \tag{19}$$

$$\bar{P}_{xy} = 2\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{Re}{M_1} + 2\right]^{-1}. \tag{20}$$

Equations (19) and (20) are solutions of the Couette problem in the absence of the transverse mass flow. With $Re/M_1 \to \infty$ (continuous medium) formulas (17), (18) are reduced to

$$\bar{u} = \frac{\exp(M_2 \bar{y} Re/M_1) - 1}{\exp(M_2 Re/M_1) - 1}, \tag{21}$$

$$\bar{P}_{xy} = \frac{\sqrt{2\pi\gamma}\, M_2 \exp(M_2 \bar{y} Re/M_1)}{\exp(M_2 Re/M_1) - 1}. \tag{22}$$

Formulas (21) and (22) correspond to those derived from the Navier-Stokes equations when we assume that there is "total adherence" of the gas at the plates.

With $Re/M_1 \to \infty$, formulas (19) and (20) reduce to (the case when there is no transverse gas flow):

$$\bar{u} = \bar{y}, \tag{23}$$

$$\bar{P}_{xy} = (2\pi\gamma)^{1/2} M_1/\mathrm{Re}. \tag{24}$$

Formulas (23) and (24) correspond to the solutions of Navier-Stokes /52 continuity equations for the case when there is no transverse gas flow. The second limiting case is the state of the free molecule flow $\mathrm{Re}/M_1 \to 0$. For this case formulas (17) and (18) are transformed into

$$\bar{u} = 0.5; \quad \bar{P}_{xy} = 1 + (\pi\gamma/2)^{1/2} M_2.$$

These results correspond to solutions of the problem by methods utilized in the region of free molecule flow.

Of considerable interest is the investigation of the influence of gas blowing and suction on the value of the frictional stresses in the rarefied gas. Using formulas (18) and (20), we obtain the following relationships, which enable us to observe the relative influence of blowing and suction on the coefficient of friction:

(a) for the plate where the gas is blown into the basic stream

$$\frac{c_{f\,\mathrm{in}}}{c_{f\mathrm{w.in}}} = \frac{\left[1 + \left(\frac{\pi\gamma}{2}\right)^{1/2} M_2\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{\mathrm{Re}}{M_1} + 2\right]}{\left[\exp(M_2 \mathrm{Re}/M_1) - 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{1}{M_2} + 1\right] + 2};$$

(b) for the plate through which there is gas suction

$$\frac{c_{f\,\mathrm{ex}}}{c_{f\mathrm{w.ex}}} = \frac{\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{\mathrm{Re}}{M_1} + 2\right]\left[1 + \left(\frac{\pi\gamma}{2}\right)^{1/2} M_2\right]\exp(M_2 \mathrm{Re}/M_1)}{\left[\exp(M_2 \mathrm{Re}/M_1) - 1\right]\left[\left(\frac{2}{\pi\gamma}\right)^{1/2} \frac{1}{M_2} + 1\right] + 2}.$$

The graphical representation of relationships (17) and (18) is shown in figure 1. The calculations are made using $\gamma = 5/3$.

Figure 2a illustrates the results of calculations of the coefficient of friction for the plate through which the gas is blown ($\bar{y} = 0$), with different speeds of blowing M_2 and different rarefaction parameters Re/M_1. The analogous

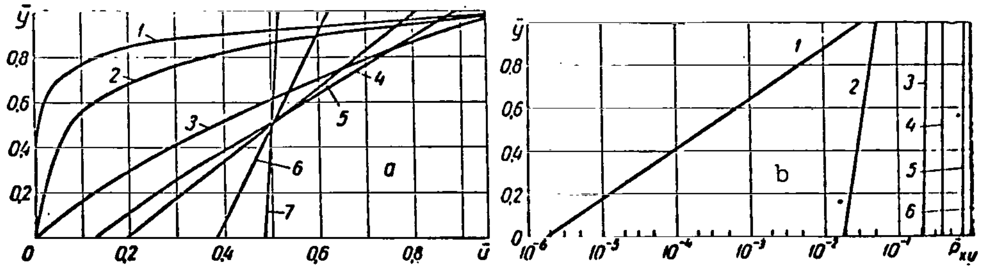

Figure 1. Velocity profiles (a) and the frictional stresses (b) for different values of the rarefaction parameter Re/M_1 with $M_2 = \bar{j}M_1 = 0.01$: a) 1 - $Re/M_1 = 1{,}000$; 2 - 500; 3 - 100; 4 - 10; 5 - 5; 6 - 1; 7 - 0.1; b) 1 - 10,000; 2 - 1,000; 3 - 100; 4 - 10; 5 - 1; 6 - 0.1.

results for the plate through which the gas is sucked ($\bar{y} = 1$), are shown in figure 2b.

Glossary of Symbols

f - molecular velocity distribution function; ξ - molecular velocity; ξ_x, ξ_y, ξ_z - xyz components of the molecular velocity; m - mass of molecule /54 R - gas constant; $\gamma = c_p/c_v$ - ratio of specific heats; λ - mean free path; μ viscosity coefficient for the Maxwell particles (ref. 1); k - Boltzmann's constant; T - gas temperature; n - number of molecules per unit volume; $\rho = mn$ - gas density; u - macroscopic gas velocity in direction x; u_0 - velocity of the moving plate (in direction x); v - macroscopic gas velocity in direction y; L - distance between the plates; j - gas mass flow in the direction y, per unit surface area; $P_{xy} = -\int(\xi_x - u)(\xi_y - v)\,d\xi$ - gas frictional stress in the plane parallel to the plate; $Re = \rho u_0 L/\mu$ - Reynold's number; $M_1 = u_0(\gamma RT)^{-1/2}$ - Mach number, for the moving wall; $M_2 = v(\gamma RT)^{-1/2}$ - Mach number, for the y component of gas velocity; $c_{f_{in}} = 2(P_{xy})_{\bar{y}} = 0/\rho u_0^2$ - coefficient of friction at the plate through which the gas is blown; $c_{f_{ex}} = 2(P_{xy})_{\bar{y}} = 1/\rho u_0^2$ - coefficient of friction at the plate through which the gas is sucked; $(c_f M_1)_0 = (2/\pi\gamma)^{1/2}$ - the product of M_1 and the coefficient of friction during free molecule flow and during the absence of transverse mass flow; $\bar{y} = y/L$; $\bar{u} = u/u_0$; $\bar{v} = v/u_0$; $\bar{u}_i = u_i/u_0$; $\bar{v}_i = v_i/u_0$; $\bar{P}_{xy} = P_{xy}(2\pi)^{1/2}/mnu_0(RT)^{1/2}$; $\bar{j} = \rho v/\rho u_0 = M_2/M_1$; $\Phi = \Phi(\xi x, \xi y, \xi z)$.

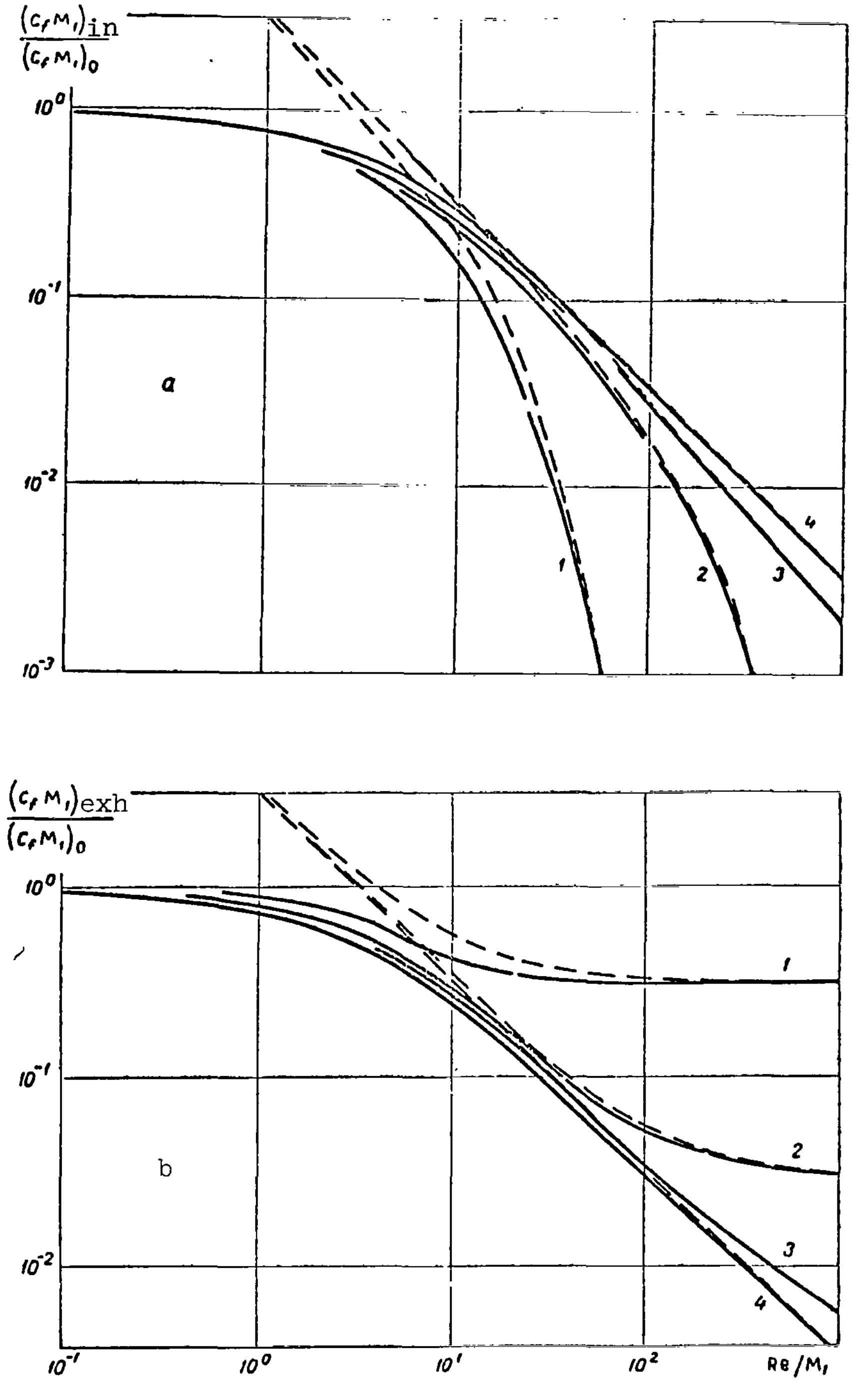

Figure 2. Coefficient of friction during blowing (a) and suction (b) as a function of the rarefaction parameter Re/M_1 (solid line - computed by using (18) with $\bar{y} = 0$ (a) and $\bar{y} = 1$ (b); broken line - computed by using Navier-Stokes continuity equations): 1 - $M_2 = 0.1$; 2 - 0.01; 3 - 0.001; 4 - 0.

Summary

On the basis of the molecular model of a gas the steady flow of rarefied Maxwell gas between two permeable surfaces moving relative to each other at a low speed is considered. Formulas (17) and (18) obtained in this work make it possible to calculate the velocity and shearing stress fields for any degree of gas rarefaction under conditions of transversal mass flow. The effect of rarefaction on the surface friction factor is studied when there is blowing and suction of gas (fig. 2). It is shown that in this case the coefficient of friction decreases with parameter Re/M_1, where $Re/M_1 \sim L/\lambda$.

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Translated for the National Aeronautics and Space Administration by John F. Holman and Co. Inc.

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